Zeeman effect

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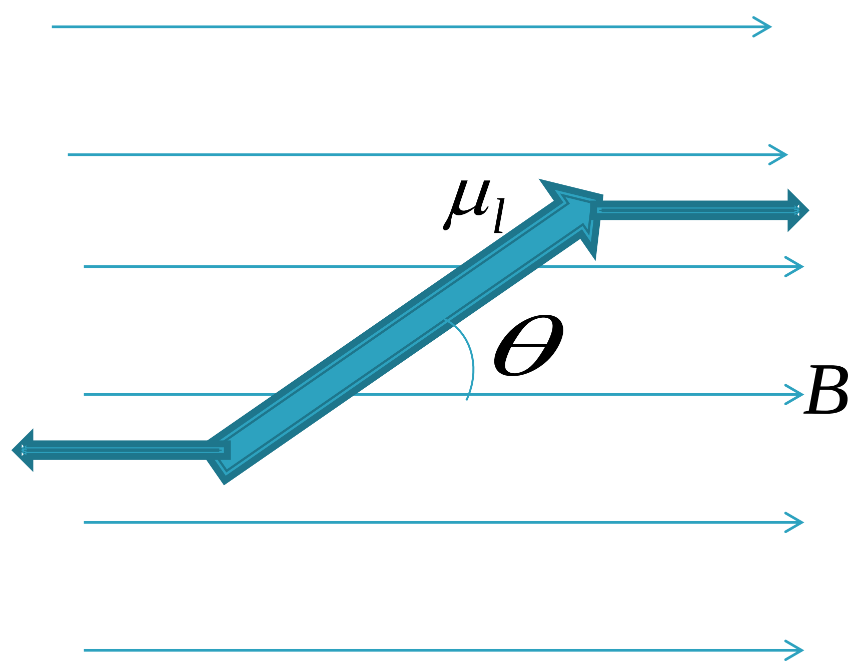
Normal Zeeman effect The splitting of a spectral line into three component in a magnetic field when viewed in a direction perpendicular to the magnetic field is called normal Zeeman effect.

$$\vec{\mu_l} = -\mu_B g_l \sqrt{l(l+1)} \quad (1)$$



$$\vec{\tau} = \vec{\mu_l} \times \vec{B}$$
$$\vec{\tau} = \mu_l B \sin\theta \quad (2)$$

Potential energy gained by the dipole from $\theta = \frac{\pi}{2}$ (minimum potential energy) to angle θ

$$V_m = \int_{\pi/2}^{\theta} \tau d\theta$$

$$V_m = \int_{\pi/2}^{\theta} \tau d\theta = \mu_l B \int_{\pi/2}^{\theta} \sin\theta d\theta = -\mu_l B [\cos\theta]_{\pi/2}^{\theta} = -\mu_l B \cos\theta$$

$$V_m = \mu_B g_l \sqrt{l(l+1)} B \cos\theta$$

Using (1) and $L\cos\theta = m_l \hbar$

$$\cos\theta = \frac{m_l \hbar}{\sqrt{l(l+1)}\hbar}$$

$$V_m = \mu_B g_l \sqrt{l(l+1)} . B \frac{m_l}{\sqrt{l(l+1)}}$$

$$\cos\theta = \frac{m_l}{\sqrt{l(l+1)}}$$

$$V_m = m_l \mu_B B$$

If no magnetic field is applied and electron jumps from higher state to lower state the frequency is given by

$$h\nu_0 = E_{0i} - E_{0f} \Rightarrow \nu_0 = \frac{E_{0i} - E_{0f}}{h}$$

In magnetic field the total energy of atom becomes

$$E = E_0 + V_m = E_0 + m_l \mu_B g_l B$$

$$E_i = E_{0i} + (m_l)_i \mu_B g_l B$$

$$E_f = E_{0f} + (m_l)_f \mu_B g_l B$$

$$E_i - E_f = E_{0i} - E_{0f} + (m_l)_i \mu_B g_l B - (m_l)_f \mu_B g_l B$$

$$\nu = \frac{E_i - E_f}{h} = \frac{E_{0i} - E_{0f}}{h} + \frac{(m_l)_i - (m_l)_f}{h} \mu_B g_l B$$

$$\nu = \nu_0 + \frac{\Delta m_l}{h}\mu_B g_l B$$

According to selection rule

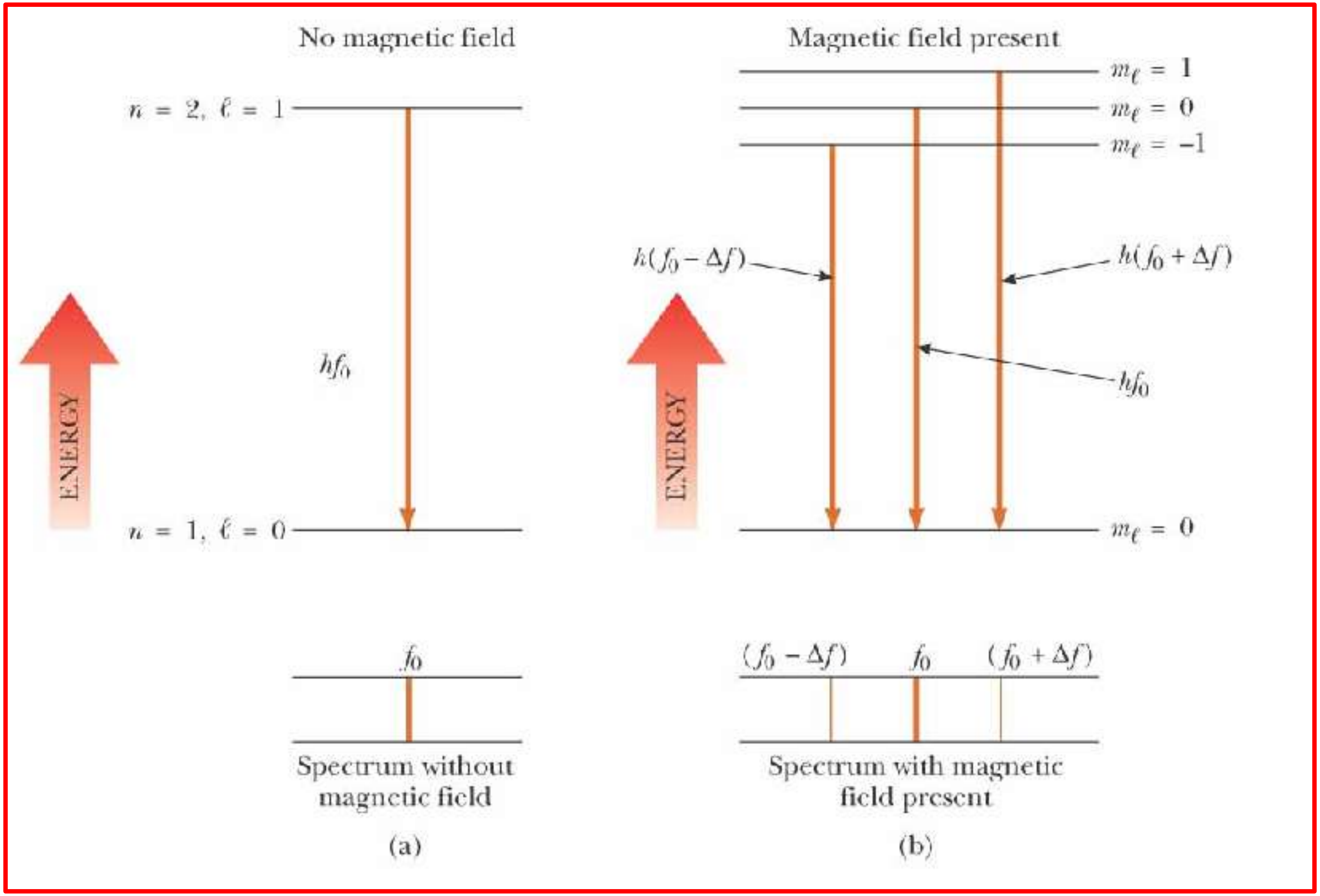
$$\Delta m_l = 1, 0, -1, and, g_l = 1$$

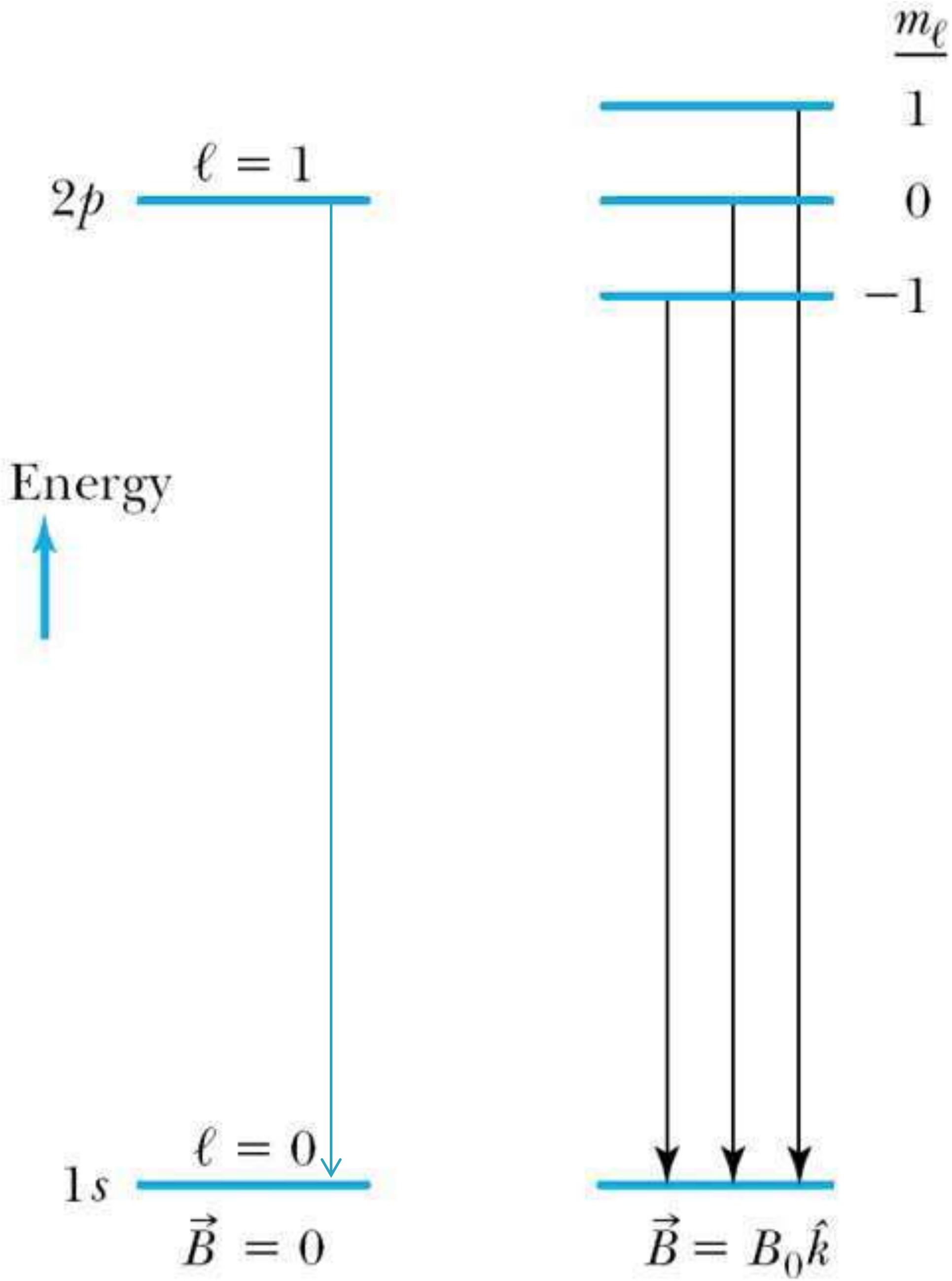
$$\nu_1 = \nu_0 + \frac{1}{h}\mu_B B \quad \Rightarrow \quad \nu_1 = \nu_0 + \Delta\nu$$

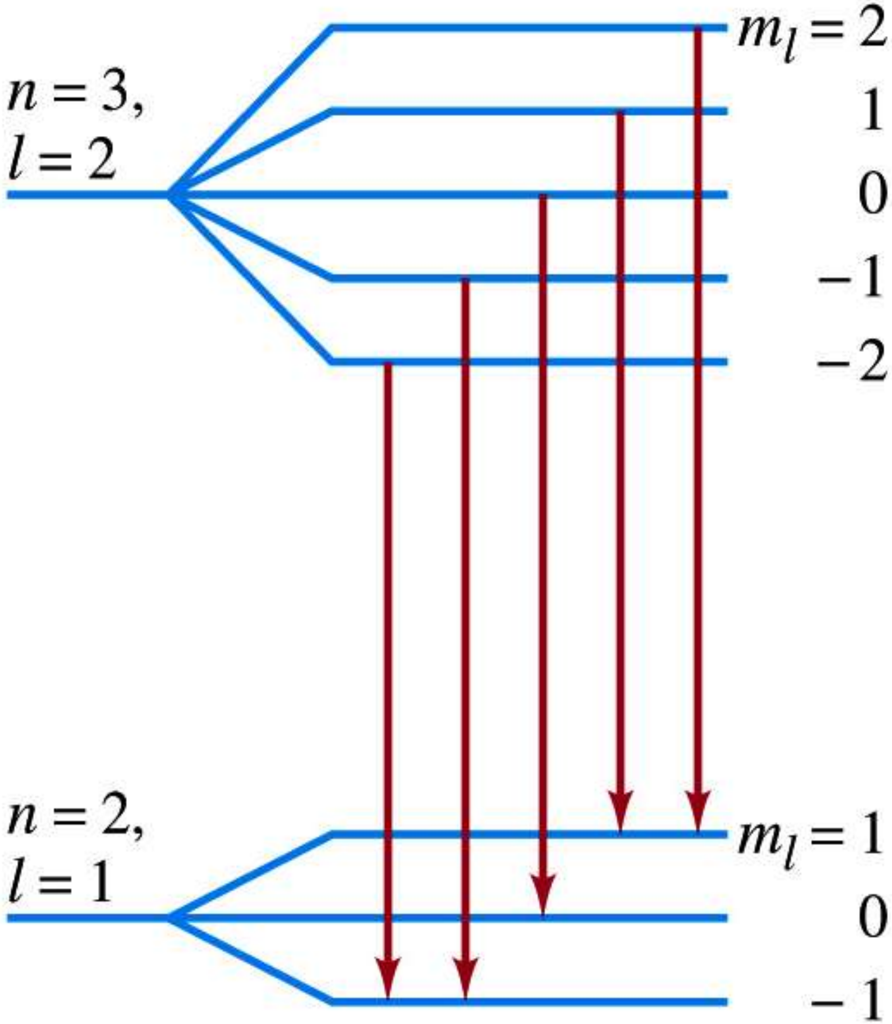
$$\nu_2 = \nu_0 \qquad\qquad \nu_2 = \nu_0$$

$$\nu_3 = \nu_0 - \frac{1}{h}\mu_B B \qquad \nu_3 = \nu_0 - \Delta\nu$$

Change in frequency $d\nu = \pm\frac{\mu_B B}{h}$


No magnetic field
Magnetic field present
n = 2, ℓ = 1
n = 1, ℓ = 0
hf0
ENERGY
mℓ = 1
mℓ = 0
mℓ = −1
mℓ = 0
h(f0 − Δf)
h(f0 + Δf)
hf0
f0
(f0 − Δf)
f0
(f0 + Δf)
Spectrum without magnetic field
Spectrum with magnetic field present
(a)
(b)


m_ℓ
1
0
−1
$\ell = 1$
2p
Energy
$\ell = 0$
1s
$\vec{B} = 0$
$\vec{B} = B_0\hat{k}$



$n = 3,$
$l = 2$
$m_l = 2$
1
0
−1
−2
$n = 2,$
$l = 1$
$m_l = 1$
0
−1

The Zeeman effect is the splitting of a spectral line by a magnetic field. The reason for the Zeeman effect is that in a magnetic field, the angular momentum quantum state can undergo a displacement from degeneracy. For example, the p orbital has three possible angular momentum quantum states that are degenerate (of the same energy) under normal circumstances. However, each angular momentum quantum state has a magnetic dipole moment associated with it, so the effect of a magnetic field is to separate the three states into three different energy levels. One state elevates in energy, one lowers in energy, and one remains at the same energy. The separation of these quantum states into three different energy levels results in 3 different excitation states with slightly different energies that give rise to three spectral lines of slightly different energy (one of the same energy as the original spectral line, one more energetic, and one less energetic) upon relaxation of the atom. This is the simplist case of the Zeeman effect, known as the Normal Zeeman effect.

Zeeman shift

For normal Zeeman effect spin S = 0, s = 0, $j = l + s, \rightarrow j = l$

$$g_j = \left[1 + \frac{j(j+1) - l(l+1) - s(s+1)}{2j(j+1)}\right]$$

$$g_j = \left[1 + \frac{l(l+1) - l(l+1)}{2j(j+1)}\right] = 1$$

$$d\nu = \pm \frac{\mu_B B}{h}$$

Also $\nu = \frac{c}{\lambda}$

$$d\nu = -\frac{c}{\lambda^2} d\lambda$$

$$\pm \frac{\mu_B B}{h} = -\frac{c}{\lambda^2} d\lambda$$

Zeeman shift $$d\lambda = \pm \frac{\lambda^2}{c} \frac{\mu_B B}{h}$$

Determination of e/m using Zeeman shift

$$d\lambda = \pm \frac{\lambda^2}{c} \frac{e\hbar B}{2mh} . \frac{2\pi}{2\pi}$$

$$d\lambda = \pm \frac{\lambda^2}{c} \frac{e\hbar B}{4\pi m\hbar}$$

$$d\lambda = \pm \frac{\lambda^2}{c} \frac{eB}{4\pi m}$$

$$\frac{e}{m} = \frac{4\pi c}{eB} d\lambda$$